FAR BEYOND

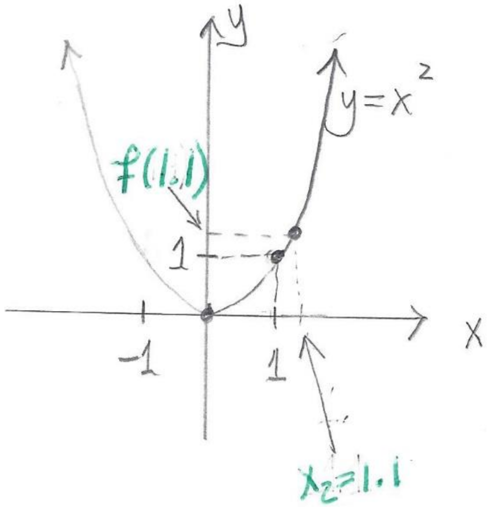
# MAT122

## Definition of the Derivative

Stony Brook University

# Finding Equation of Tangent Line

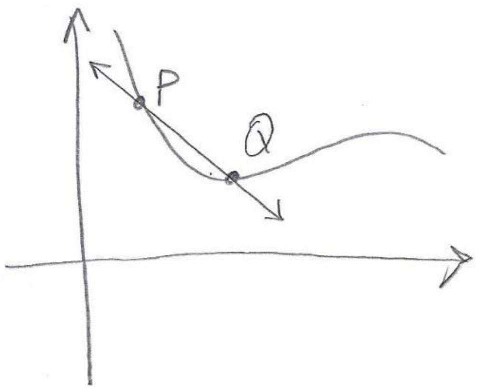
ex. Find the equation of the tangent line to the parabola $y = x^2$ at the point $P_1$: (1,1)




$\underline{x_2}$ $\underline{m}$

# Average vs Instantaneous Velocity

Suppose a ball is dropped from upper deck of CN Tower, 450m above the ground.
Find the velocity of the ball after 5 seconds. Use the model $s(t) = 4.9t^2$

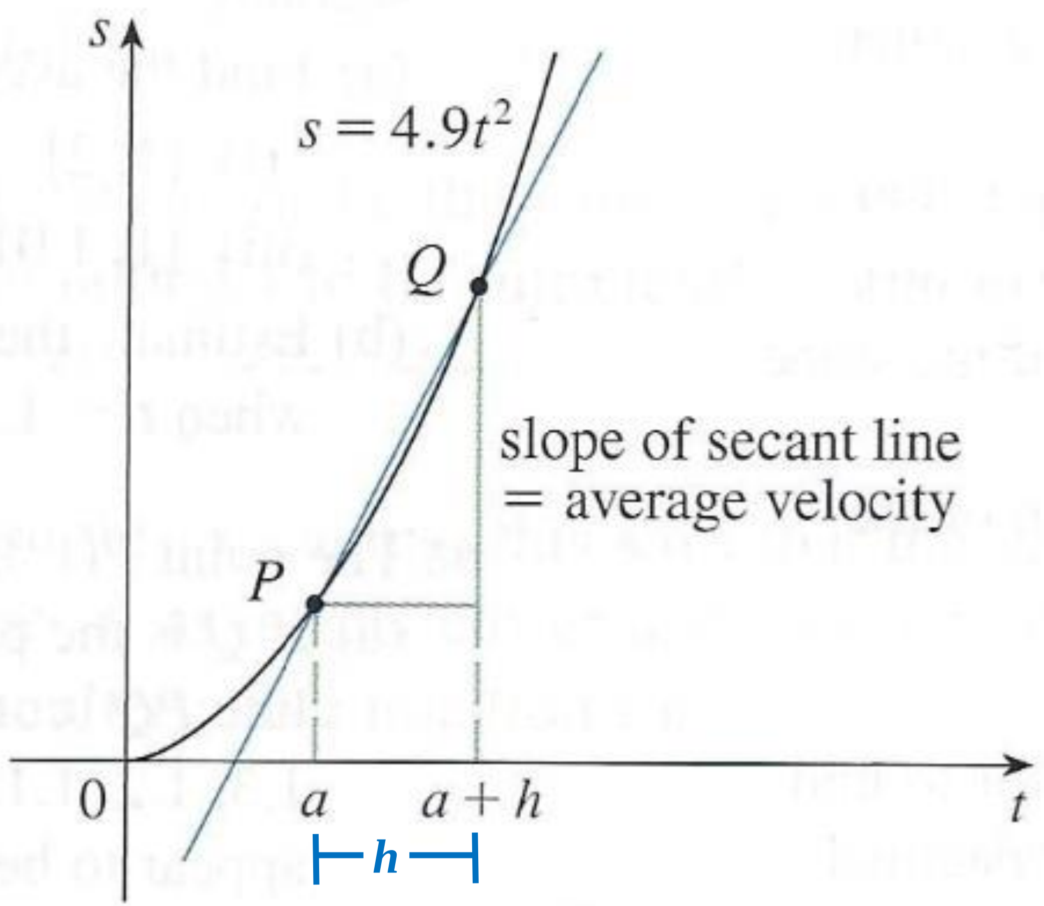


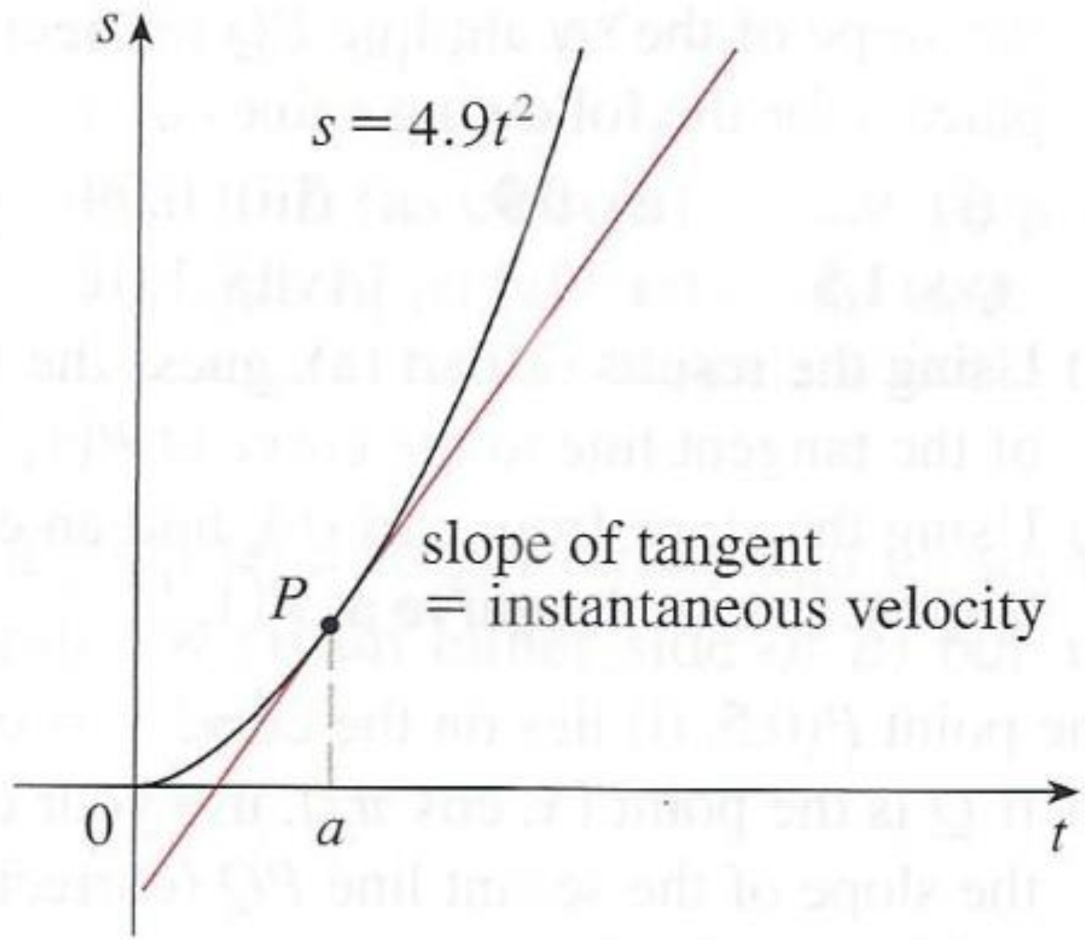

Use a **LIMIT** to make distance infinitely small

# Instantaneous Velocity

**General Formula**

$$v_{inst} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



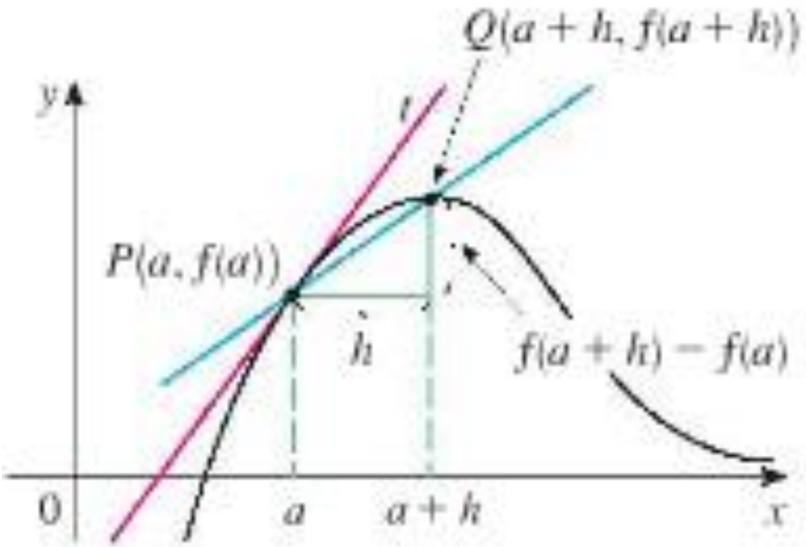



# Rate of Change

The slope of a tangent line at a point on a curve measures the **rate of change** at that point

Definition uses the difference quotient:

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



# Rate of Change (cont'd)

ex. Find the slope of the tangent line to $y = 4x - x^2$ at $a = 1$ using the definition of rate of change.

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Follow up question:

Find equation of this tangent line.

# Definition of the Derivative - Formula

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

ex: Find the derivative of $f(x) = x^2 - 8x + 9$ at $x = a$

$$= \boxed{2a - 8}$$

# General Definition of the Derivative

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \Rightarrow f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

ex: Find the general derivative of $f(x)=x^3-x$

$$f'(x)=\boxed{3x^2-1}$$

# General Definition of the Derivative - Do

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Do: Use the definition of the derivative to find the derivative of:

$$f(x) = 1 - 3x^2$$

$$= \boxed{-6x}$$

# General Definition of the Derivative – Example 2

ex: Find the general derivative of $f(x)=\sqrt{x}$. State the domain of $f\,'$.

$$f\,'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f(x+h)=\sqrt{x+h}$$

$$f\,'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

Domain:

$$=\boxed{\frac{1}{2\sqrt{x}}}$$

# General Definition of the Derivative – Example 3

ex: Find the general slope of the tangent line of $f(x) = \dfrac{1-x}{2+x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Do: $f(x+h)$

$$f'(x) = \boxed{-\frac{3}{(2+x)^2}}$$

# Second Derivative Example

Recall: the derivative of $f(x) = x^3 - x$ : $f'(x) = 3x^2 - 1$

To find $f''(x)$, take the derivative of $f'(x)$.

Do: find $f'''(x)$ and $f^{(4)}(x)$.

$$\therefore \boxed{6x = f''(x)}$$

# Derivative Exceptions

a function is <u>not</u> **differentiable** where there is a:

1. corner
2. discontinuity
3. vertical tangent